Winter Contest 2023

Solutions presentation

January 28, 2023

Winter Contest 2023 Jury

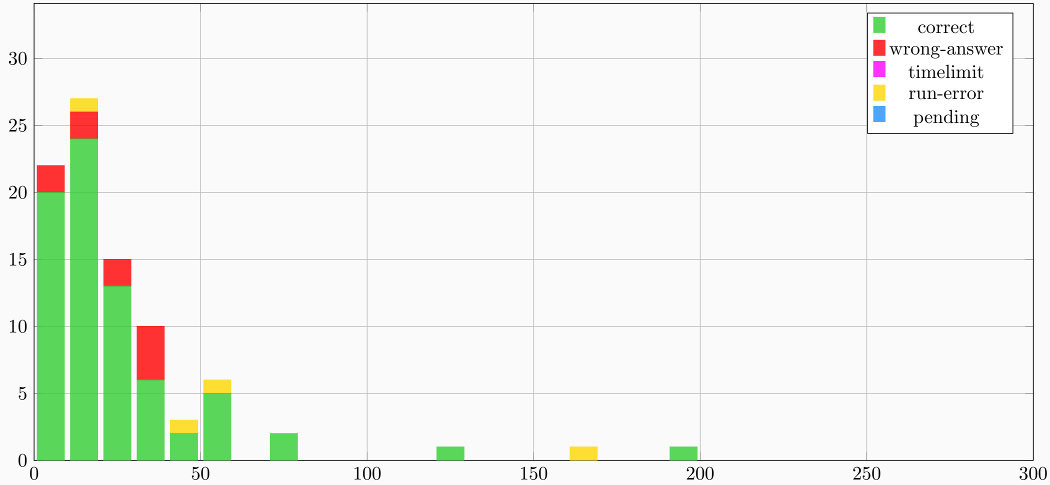
- **Florian Kothmeier**
  Friedrich–Alexander University Erlangen–Nürnberg
- **Felicia Lucke**
  CPUlm
- **Jannik Olbrich**
  CPUlm
- **Christopher Weyand**
  Karlsruhe Institute of Technology
- **Marcel Wienöbst**
  University of Lübeck
- **Wendy Yi**
  Karlsruhe Institute of Technology
- **Michael Zündorf**
  Karlsruhe Institute of Technology

Big thanks to our test solvers

- **Niko Fink**
  University of Passau
- **Michael Ruderer**
  CPUlm
- **Erik Sünderhauf**
  Technical University of Munich

I: Infinity Issues

Problem Author: Michael Zündorf



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Solution

- Read the complete line containing the text
- Print it character for character
- If the position $i = 0 \bmod w$ print in addition a newline
  except if $i = 0$

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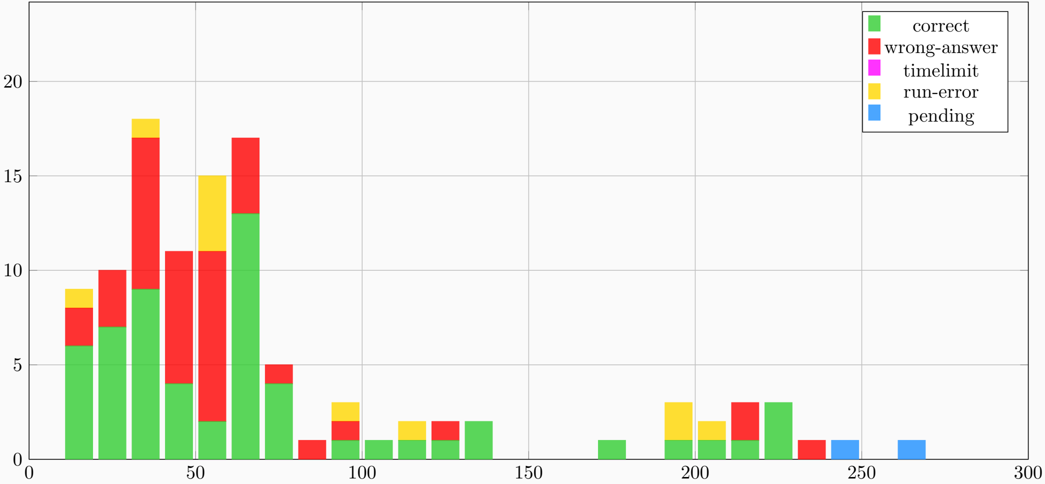
Tipps for Common Errors

If you combine `std::cin` and `std::getline` make sure that you read the `'\n'` character ending the previous line

C: Christmas Calories

Problem Author: Jannik Olbrich



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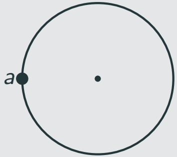

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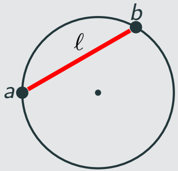

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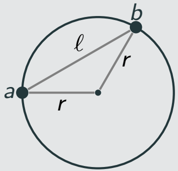

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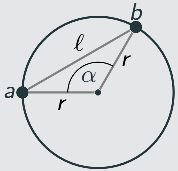
- Compute the angle α: $\ell^2 = r^2 + r^2 - 2 \cdot r \cdot r \cdot \cos\alpha$ (law of cosines)



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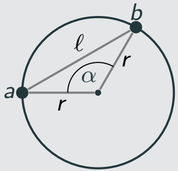
- Answer is $1 - \alpha/\pi$



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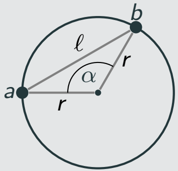

Alternative solution: Binary search over α or e.g. the x-coordinate of b

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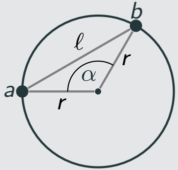

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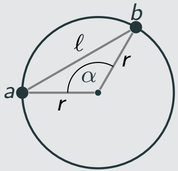

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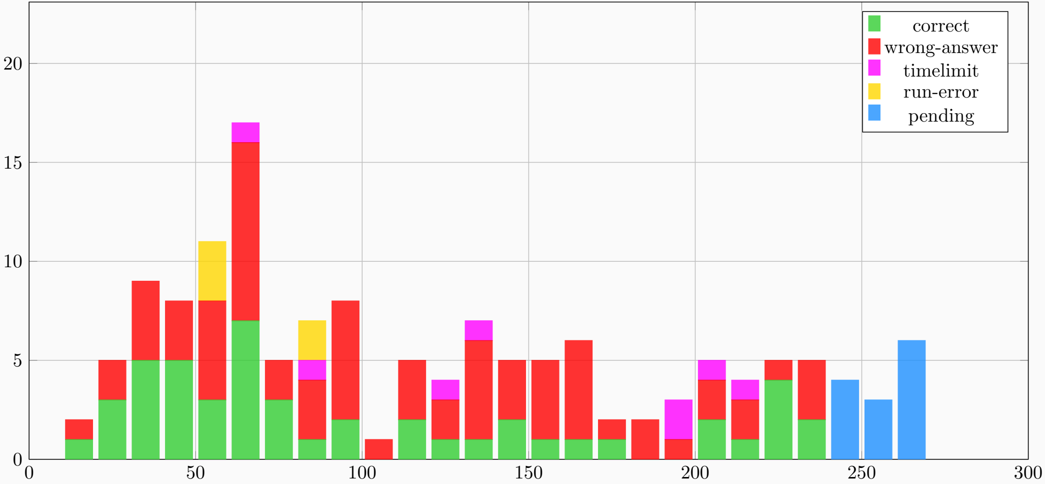
Possible pitfalls

- `float` is too imprecise
- Edge case $\ell > 2r$ may result in `NaN`

J: Jinxed Jewelry

Problem Author: Michael Zündorf



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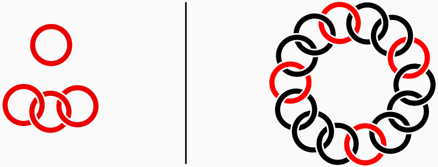

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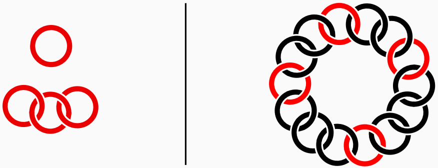

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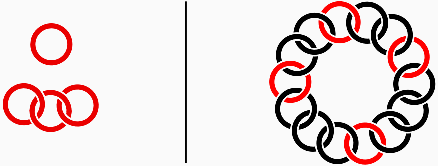

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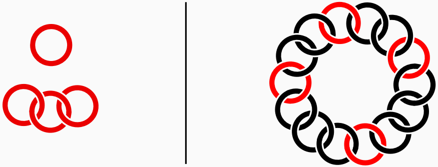

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- You need to open a chain links such that you end up with $b \leq a$ chains remaining
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- If you open chain links from the shortest chain you have the chance to completely use up a chain
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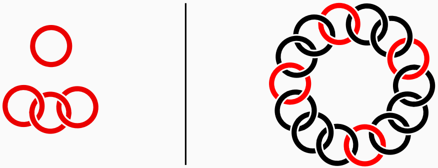

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$\Rightarrow$ It is optimal to open chain links from the shortest chains first

D: Discus Domination

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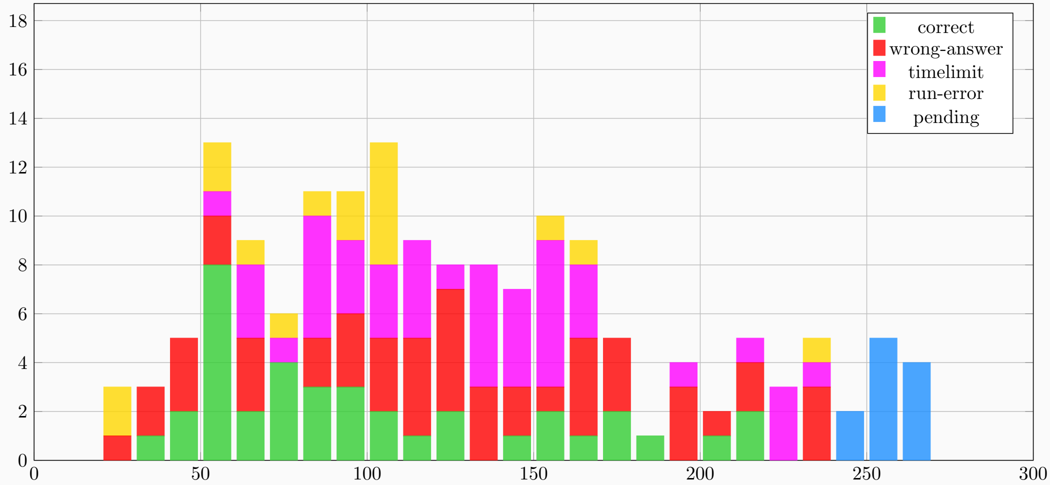

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Given an integers $a_1, \ldots, a_n$, maximise $a_j - a_i$, where $0 \leq j - i \leq m$ $(1 \leq n, m \leq 10^9)$

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  - Can be queried in $\mathcal{O}(\log m)$ using a min-heap.
    - C++: use `std::multiset` or `std::map`
    - Java: use `java.util.TreeMap`
    - Python: `from queue import PriorityQueue`

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    - C++: use `std::multiset` or `std::map`
    - Java: use `java.util.TreeMap`
    - Python: `from queue import PriorityQueue`
  - For $j = 1 \to n$:
    - insert a_j into the (multi) set.
    - query $a_i = min(set)$.
    - delete a_{j-m} from the set.

  $\Rightarrow \mathcal{O}(n \cdot log\ m)$

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Possible Pitfall

- Be careful of duplicate elements, e.g. use `std::multiset` instead of `std::set`.

$\Rightarrow$ If not available (e.g. in Java), use a map (e.g. `TreeMap`) and count their occurrences. Delete entries only when they appear 0-times in the map.

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- Use a `Deque` for $\mathcal{O}(1)$ insertion and deletion from both ends.

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Alternative Solution

- Use a `Deque` for $\mathcal{O}(1)$ insertion and deletion from both ends.
- Add the current value and position (a_j, j) at the end and remove the preceding entry while its value is higher. $\Rightarrow$ Smallest element will always be the first

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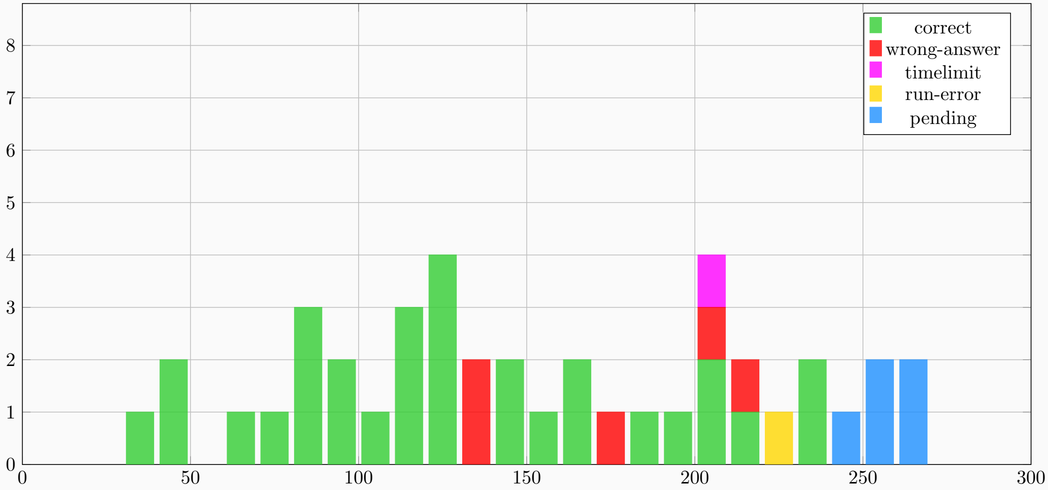
- Remove elements from the front when their position is less than $j - m$
- Each element will be added once (and deleted once) from the list $\Rightarrow \mathcal{O}(n)$

E: Elegant Exterior

Problem Author: Marcel Wienöbst



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- Ternary search over w/h to find the optimal ratio of width and height. For a fixed ratio, one can compute w and h and thus the maximum area by binary search.

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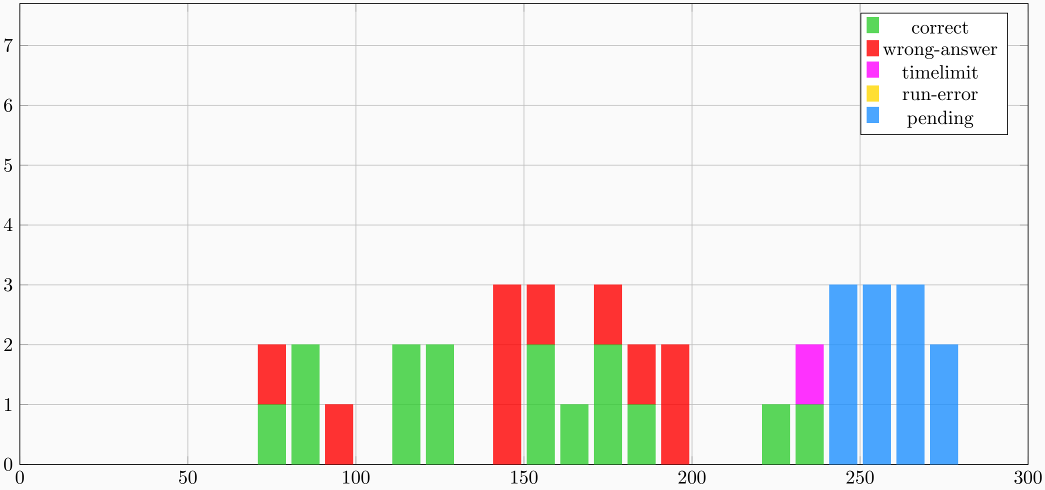
- This is fast enough. However, there is a simpler solution.
- There is an optimal ratio of w and h independent of n (around 1.2221, but that's not even necessary to know). Thus, the maximal area scales with n^2 and the answer is simply $0.0185303 \cdot n^2$, where 0.0185303 is the solution to Sample Input 1.

G: Gorgeous Garment

Problem Author: Wendy Yi



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You are given

- the number of stitches of each round (which are increasing)
- and the amount and order of the colours.

How many rounds can you crochet such that each colour stripe is at least as wide as the last?

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- If you can crochet i rounds of the pattern, you can crochet fewer rounds as well.
- Use binary search to determine the maximum number of rounds.

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- Test if it is possible to crochet i rounds:
  - Colour stripes are wider towards the outer edge
    $\Longrightarrow$ use outermost colour for as many rounds as possible.

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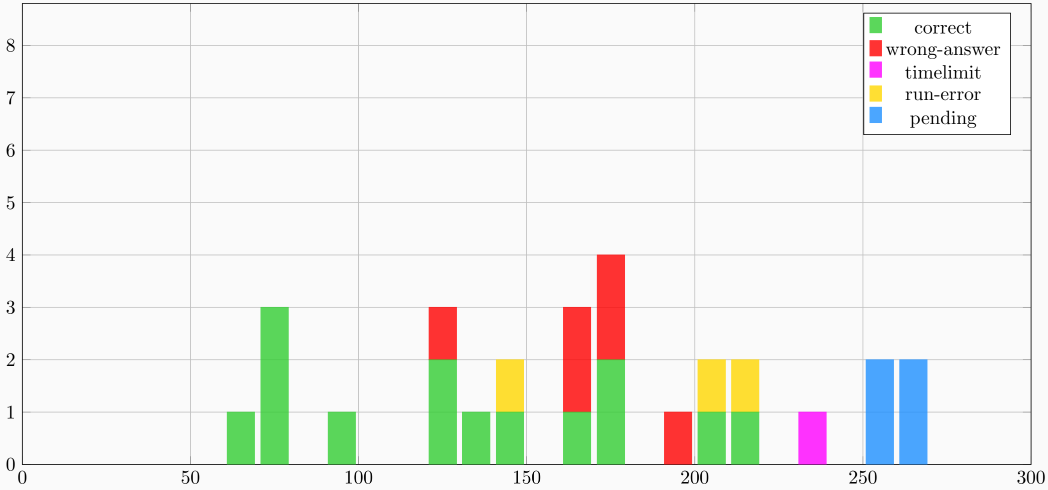
Running time: $\mathcal{O}(n \log(n))$

L: Legendary Lanparty

Problem Author: Michael Zündorf



L: Legendary Lanparty

Problem Author: Michael Zündorf

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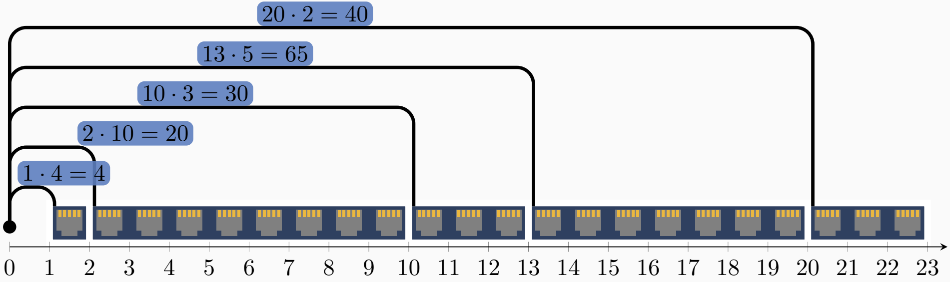
Given n tuples (c_i, m_i), reorder them such that the following sum is minimized

$$\sum_{i=1}^{n} c_i \cdot \sum_{j=1}^{i-1} m_j \ .$$



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- Observe that swapping two adjacent tuples changes the cost by

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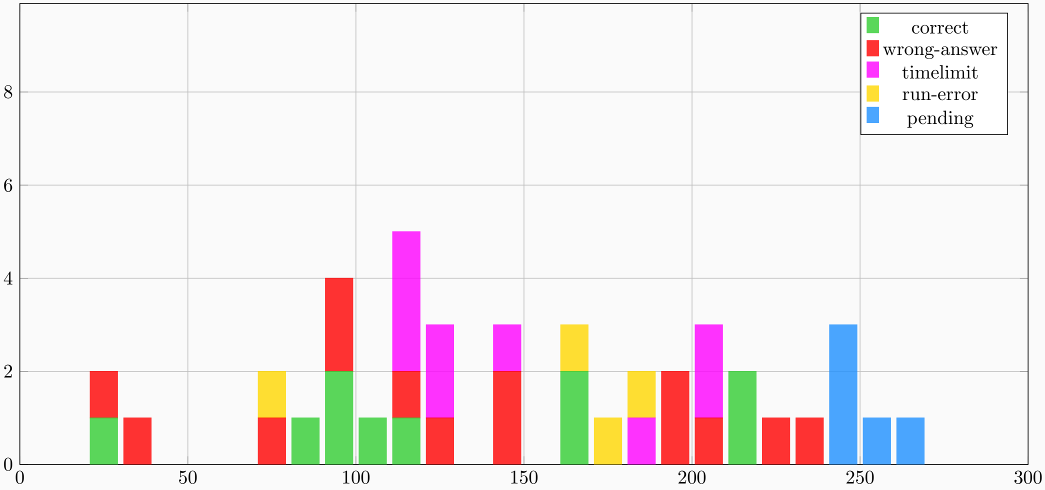
- This already implies a total order

$\Rightarrow$ We can sort by δ, and compare non adjacent elements with it

A: Alien Attack

Problem Author: Christopher Weyand



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Given an undirected, connected graph. Each time step the following happens:

- the vertex of highest degree (id as tiebreaker) is deleted. Vertex 1 is never deleted.
- any vertex that cannot reach vertex 1 is deleted.

How many steps until only vertex 1 remains?

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- consider an alternative process that deletes the highest degree node each step (nothing else)

A: Alien Attack

Problem Author: Christopher Weyand

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Given an undirected, connected graph. Each time step the following happens:

- the vertex of highest degree (id as tiebreaker) is deleted. Vertex 1 is never deleted.
- any vertex that cannot reach vertex 1 is deleted.

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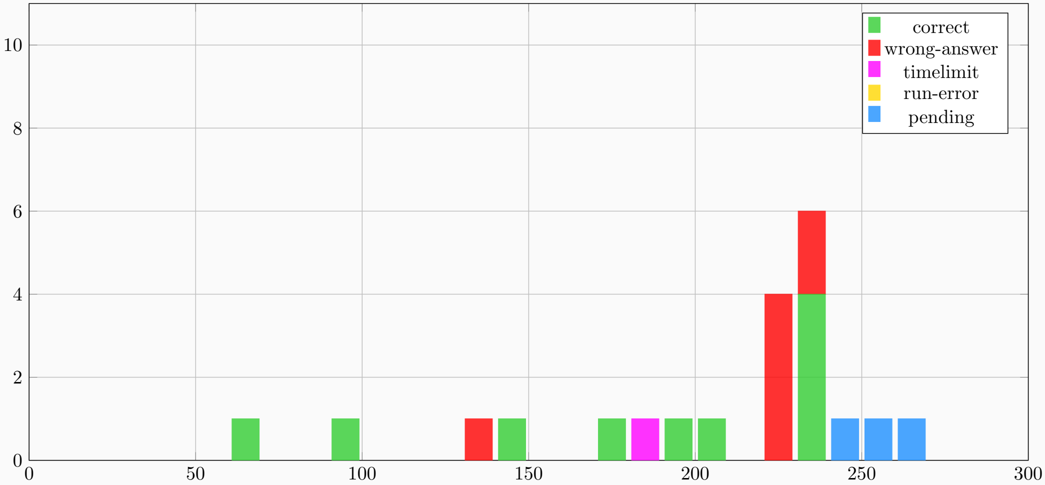
- deletion order can be computed by maintaining degrees with a priority queue in $O(m \log n)$
- by simulating the process in reverse (deletions become insertions) reachability checks can be done with a union-find data structure

M: Massive Mountains

Problem Author: The Winter Contest Jury, Julian Baldus



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Given a weighted, directed graph with red and blue edges. A and B want to get from vertex 1 to vertex n. They are not allowed to use an edge of the same colour at the same time. How long does it take them at least to get to vertex n.

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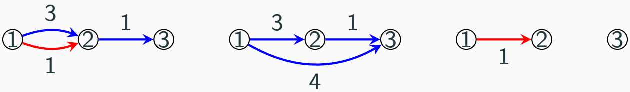

Solution

- Assume A starts with a red edge and B with a blue one.
- When A and B swap colours they both have to be on a vertex.
- Between swapping colours A and B walk through the subgraph with red/blue edges.
- We may assume that they use only shortest paths. (They are allowed to wait.)
- Step 1: Compute all shortest paths in the subgraph with red/blue edges. (Floyd Warshall)

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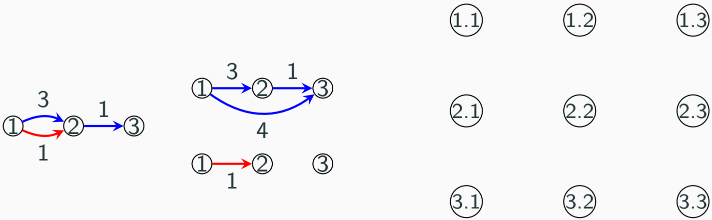

Solution (continued)

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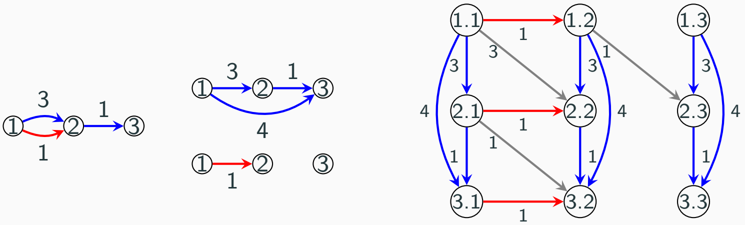

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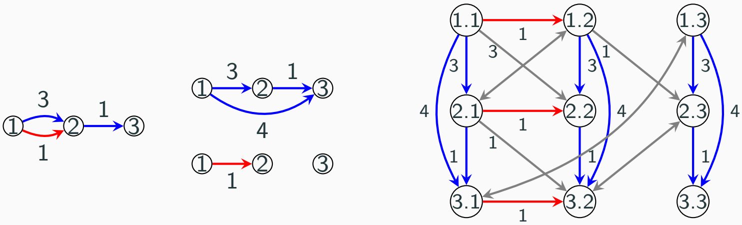

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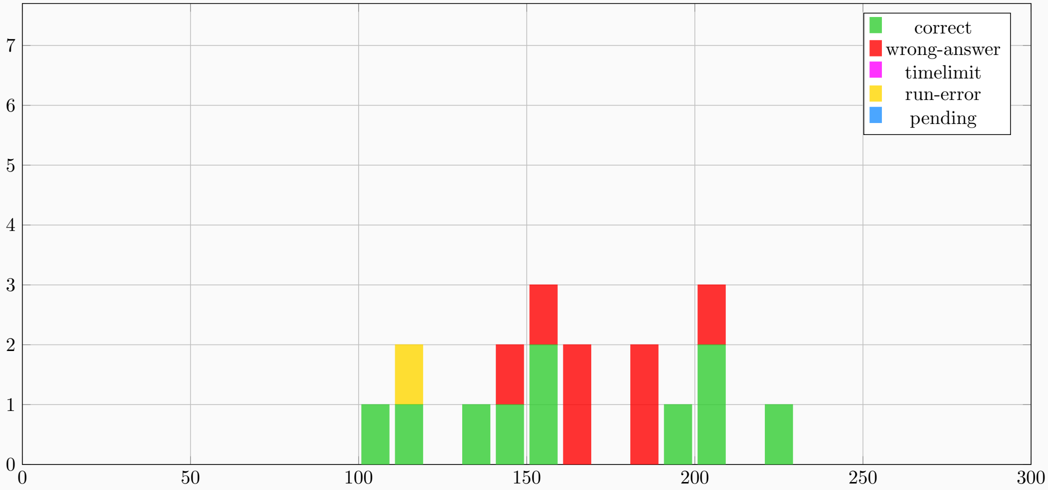
  - A and B may swap colours. Add a bidirectional arc $((r, b), (b, r))$ with cost 0.
- Find a shortest path from $(1, 1)$ to (n, n) in G' (e.g. with Dijkstra).

K: K.O. Kids II

Problem Author: Marcel Wienöbst



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Given probabilities $a_1, \dots, a_k$ of overcoming an unbeaten obstacle (already beaten obstacles are overcome every time) and a queue of n participants, calculate the maximum probability to be the first finisher.

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Solution

- Main Idea: For participant i, compute the probability of making it up to obstacle j and failing there:

$$P(i,j) = \sum_{k \leq j} P(i-1,k) \cdot \prod_{k \leq l < j} a_l \cdot (1 - a_j).$$

  In words, multiply the probability for each possible position of the previous participant by the probability to make it from there exactly to obstacle j and not further.

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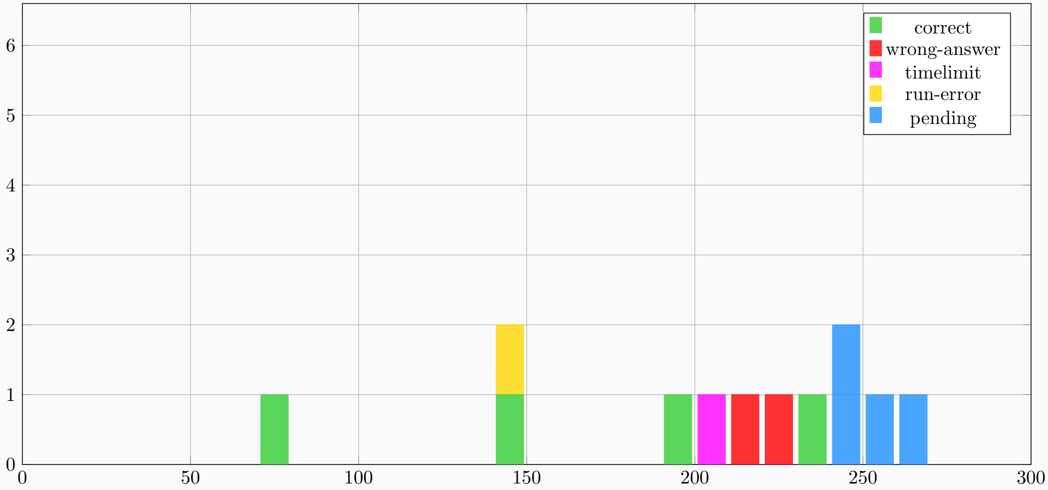
- Evaluating this naively takes time $O(nk^2)$, which is too slow.
- Instead, dynamically build up the term $T(i,j) = \sum_{k \leq j} P(i-1,k) \cdot \prod_{k \leq l < j} a_l$. It holds that $T(i,j) = (T(i,j-1) + P(i-1,j)) \cdot a_j$ and clearly $P(i,j) = T(i,j-1) \cdot (1 - a_j)$.
- This can be implemented in $O(nk)$ time.

H: Hungry Hunting

Problem Author: The Winter Contest Jury, Julian Baldus



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Given n item types with values $c_1, \ldots, c_n$. If we double c_i, how many items do we have to take to obtain a value of exactly w? Print the answer for every i.

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Solution

- Without doubling this problem is the classic coin change problem: Let $dp_\ell(k, j)$ be the minimum number of items that have total value j, given that only types $1, \dots, k$ are allowed
- For each i double the value c_i and do the classic coin change DP

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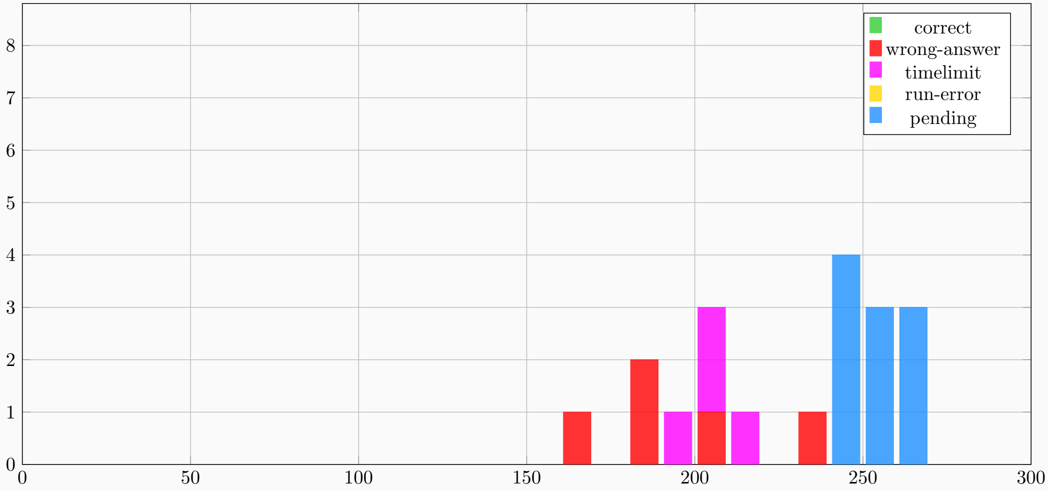
- For each i double the value c_i and do the classic coin change DP $\Rightarrow \mathcal{O}(n^2 \cdot w) \Rightarrow$ **Too slow!**
- Insight: $dp_\ell(k, j)$ for $k < i$ is independent of whether c_i is doubled or not.
- The same property holds when the DP works from the other direction: Only types $k, \dots, n$ are allowed for $dp_r(k, j)$; c_i is irrelevant for $k > i$.
- For each i, compute $double(i, j) = \min\{double(i, j - 2c_i), dp_\ell(i - 1, j)\}$:
  "number of items with total value j, given that only types $1, \dots, i$ are allowed and c_i is doubled"
- For each i, find $\min_j\{double(i, j) + dp_r(i + 1, w - j)\}$.

Total time complexity: $\mathcal{O}(n \cdot w)$

B: Broken Borders

Problem Author: Jannik Olbrich



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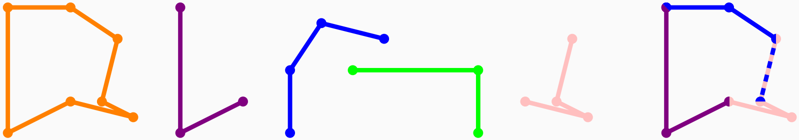
Given a simple polygon and many polylines. Can the polylines can be aligned to the polygon such that every line segment of the polygon is covered? Polylines can be used arbitrarily often.



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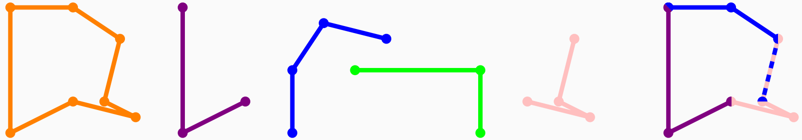

Solution

- A polyline can be aligned to a part of the polygon iff
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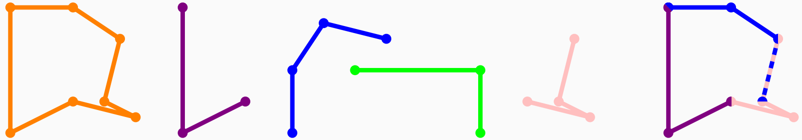

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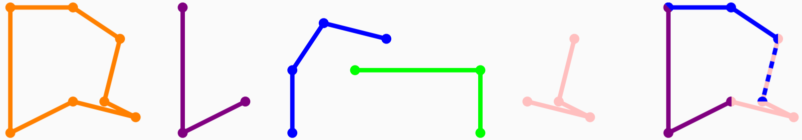

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Solution (cont.)

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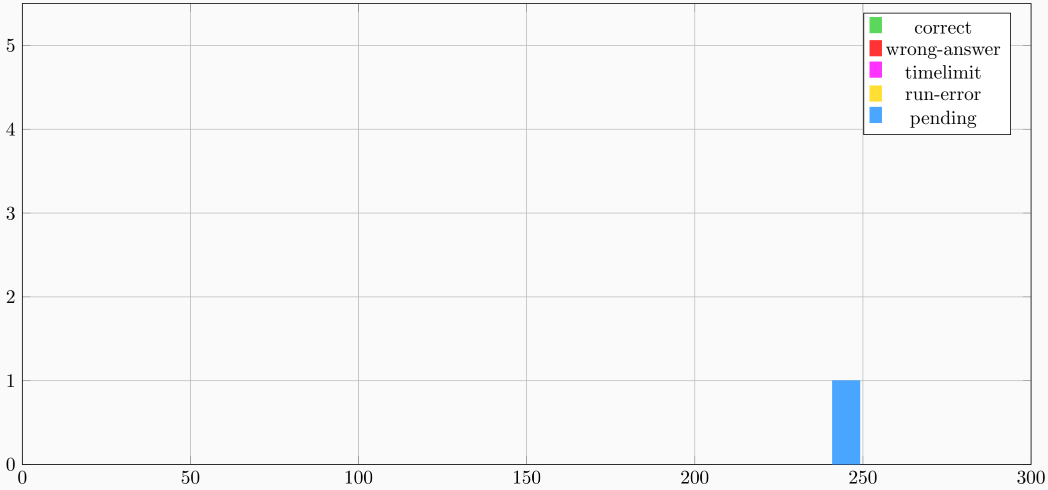
Possible pitfalls

- You need integer-save angle comparison, (`long`) `double` is not precise enough
- $\mathcal{O}(n \log^2 n)$ suffix array construction may be too slow
- $\mathcal{O}(n^{1.5})$ hashing solutions can be too slow

F: Fragmented Floor

Problem Author: Jannik Olbrich



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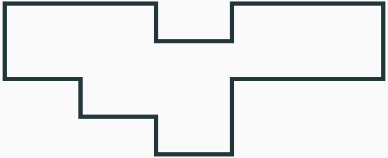

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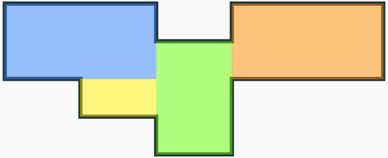

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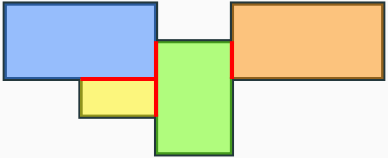

Solution

- Any solution (i.e. list of rectangles) is characterised by the axis-parallel diagonals that split the polygon into the rectangles. Call those *dissection edges*
- Number of rectangles is $1 + \#$dissection edges

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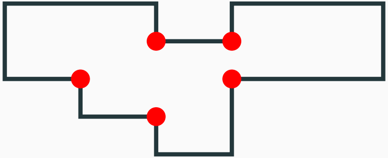

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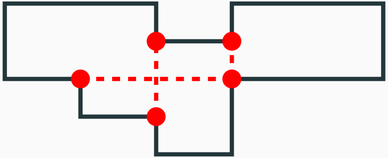

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- Number of rectangles is $1 + \#\text{dissection edges}$
- Each concave corner of the polygon must be met by (at least) one dissection edge
- A dissection edge meets at most two concave corners. Call edges incident to two corners *critical*
- Number of rectangles is $1 + \#\text{concave corners} - \#\text{used non-intersecting critical edges}$

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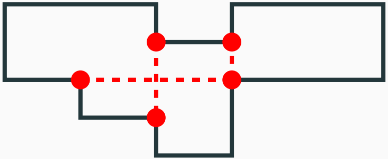

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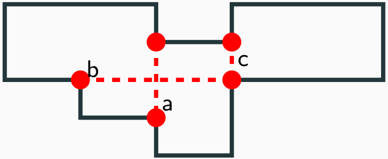

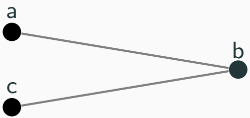

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  $\Rightarrow$ Max. set of critical edges $\widehat{=}$ max. independent set

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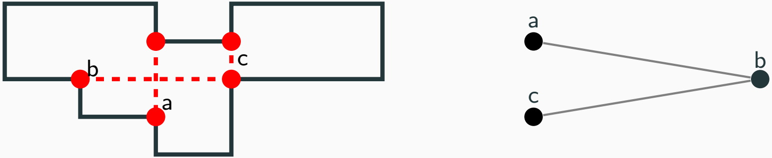

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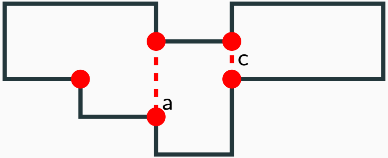

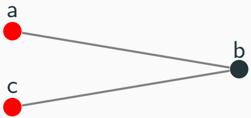

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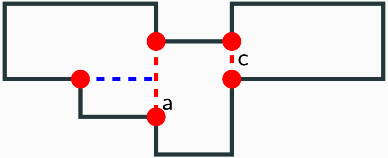
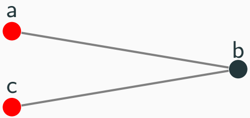


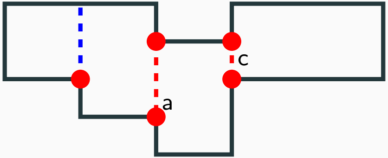


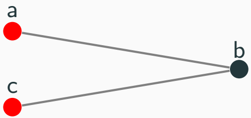
Solution (cont.)

- Number of rectangles is $1 + \#\text{concave corners} - \#\text{used non-intersecting critical edges}$
  $\Rightarrow$ Maximise number of non-intersecting critical edges
- Build graph: nodes are critical edges; connect nodes iff edges intersect
  $\Rightarrow$ Max. set of critical edges $\widehat{=}$ max. independent set
- Only horizontal and vertical edges intersect $\Rightarrow$ Graph is bipartite
  $\Rightarrow$ solve using bipartite matching (Kőnig's theorem)
- (To complete the dissection, draw a chord from each remaining convex vertex; the direction does not matter.)

F: Fragmented Floor

Problem Author: Jannik Olbrich





Solution (cont.)

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Random facts

Jury work

- 263 commits

Random facts

Jury work

- 263 commits
- 369 secret test cases ($\approx$ 28 per problem)

Random facts

Jury work

- 263 commits
- 369 secret test cases ($\approx$ 28 per problem)
- 110 jury solutions

Random facts

Jury work

- 263 commits
- 369 secret test cases ($\approx$ 28 per problem)
- 110 jury solutions
- The minimum number of lines the jury needed to solve all problems is

$$39 + 67 + 4 + 8 + 1 + 52 + 28 + 23 + 4 + 17 + 16 + 14 + 25 = 298$$

  On average 23 lines per problem